# MTAT.05.125 Introduction to Theoretical Computer Science

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Practice session 9.

## Practise session

**1.** Define $L = \{0^{2^n} \mid n \in \mathbb{N}\}$, i.e. collection of all strings of zeros whose length is a power of 2. We want to construct TM $M$ that decides $L$.

We start with implementation-level description.On input $w$, the machine $M$:

1. Moves from left to right by crossing out every second zero.
2. If in step 1 the tape contains a single zero – accept.
3. If the tape contains add number of zeros, which is $> 1$, rejects.
4. Return the head to the beginning of the tape and goto 1.

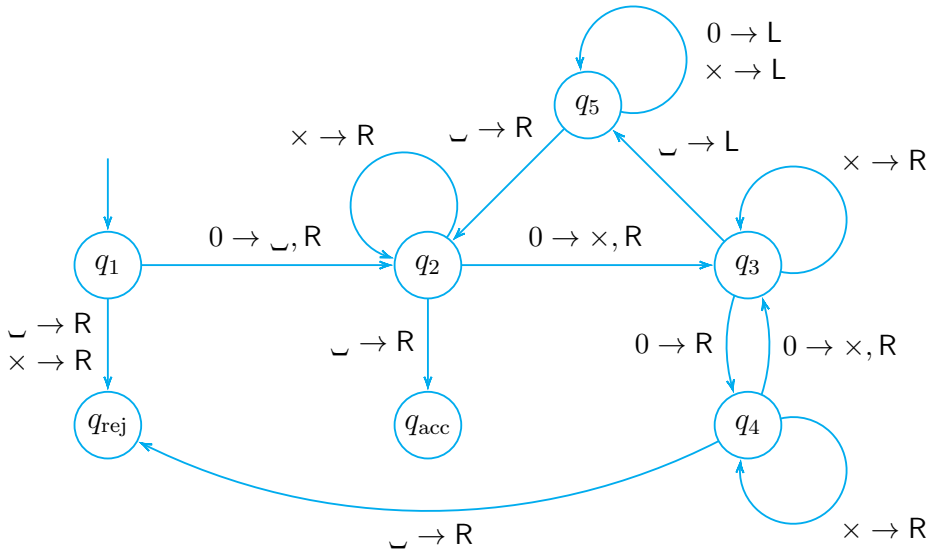
Note that if the number of zeros is $2^n$, then on each iteration it decreases by a factor of 2, and after $n$ iterations $M$ accepts.

If it is not a power of 2, then at some iteration condition of the step 3 holds and we reject.

Now we give a formal description. We build TM $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{acc}}, q_{\text{rej}})$, where

- $Q = \{q_1, q_2, q_3, q_4, q_5, q_{\text{acc}}, q_{\text{rej}}\}$;
- $\Sigma = \{0\}$;
- $\Gamma = \{0, \times, \textvisiblespace\}$;

- start state is $q_1$;
- accept state is $q_{\text{acc}}$;
- reject state is $q_{\text{rej}}$;
- transition function $\delta$ is as follows:




Here notation $0 \to \sqcup, R$ means that by reading 0 $M$ writes $\sqcup$ and moves the head to the right (in transition from $q_1$ to $q_2$). Hence, $\delta(q_1, 0) = (q_2, \sqcup, \mathsf{R})$.

Note. $M$ writes $\sqcup$ over the left-most 0 on the tape to mark the left-hand end of the tape.

Let us see the run of $M$ on input 0000:

$$
\begin{array}{ll}
q_1\,0\,0\,0\,0\,\sqcup & \sqcup\times\times q_3\times\sqcup \\
\sqcup q_2\,0\,0\,0\,\sqcup & \sqcup\times\times\times q_3\sqcup \\
\sqcup\times q_3\,0\,0\,\sqcup & \sqcup\times\times q_5\times\sqcup \\
\sqcup\times 0\,q_4\,0\,\sqcup & \sqcup\times q_5\times\times\sqcup \\
\sqcup\times 0\times q_3\sqcup & \sqcup q_5\times\times\times\sqcup \\
\sqcup\times 0\,q_5\times\sqcup & q_5\sqcup\times\times\times\sqcup \\
\sqcup\times q_5\,0\times\sqcup & \sqcup q_2\times\times\times\sqcup \\
\sqcup q_5\times 0\times\sqcup & \sqcup\times q_2\times\times\sqcup \\
q_5\sqcup\times 0\times\sqcup & \sqcup\times\times q_2\times\sqcup \\
\sqcup q_2\times 0\times\sqcup & \sqcup\times\times\times q_2\sqcup \\
\sqcup\times q_2\,0\times\sqcup & \sqcup\times\times\times\sqcup q_{\text{acc}}
\end{array}
$$