

**Midterm exam – retake**

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Student name: \_\_\_\_\_

Student ID: \_\_\_\_\_

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
<b>Total</b>	

**Question 1** (48 points).

The group of 5 students prepares for a bicycle trip. There are 8 different types of bicycles in the storehouse. Each student chooses one bicycle.

- (a) How many ways are there to choose the bicycles?
- (b) The same as (a), but it is known that the students choose bicycles of 5 different types.

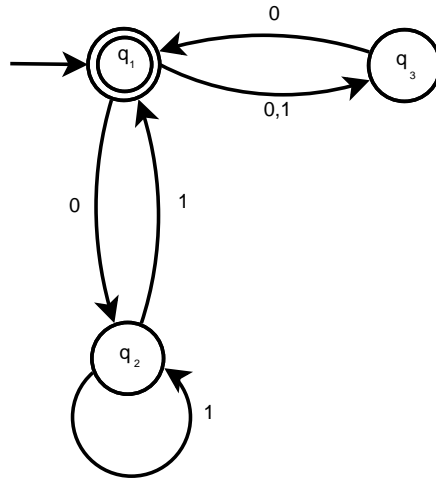
Now, the group of  $2n$  students prepares for a bicycle trip,  $n \in \mathbb{N}$ .

- (c) The students plan to stay overnight in  $n$  different tents. Each tent accommodates 2 students. How many ways to divide students into tents are there?
- (d) The same as (c), but now the tents are identical.
- (e) Each of the  $2n$  students has a bicycle. The students are divided into pairs, as before. How many ways to arrange the bicycles to the students are there, if no student receives his own bicycle or the bicycle of his/her friend (with whom he/she shares a tent)?



**Question 2** (24 points).

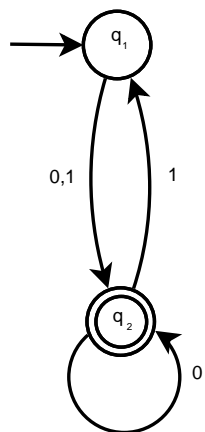
Convert the following nondeterministic finite automaton into an equivalent deterministic automaton. Show all the steps in the conversion process.





**Question 3** (24 points).

Construct a regular expression for the language  $\mathcal{L}$  defined by the following deterministic finite automaton:



Show all the steps in the algorithm.



**Question 4** (24 points).

- (a) Prove that the following language is not regular:

$$\mathcal{L}_1 = \{ (01)^n(10)^n \mid n \in \mathbb{N} \} .$$

- (b) Is the following language regular or not? Justify your answer.

$$\mathcal{L}_2 = \{ (01)^n(01)^n \mid n \in \mathbb{N} \} .$$

Remark: please note that the definitions of the languages  $\mathcal{L}_1$  and  $\mathcal{L}_2$  differ by the order of the symbols '0' and '1' within the second parentheses of the corresponding regular expressions.





