

Midterm exam

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Student name: _____

Student ID: _____

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 110 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (40 points).

A group of n students goes for a weekend trip.

- (a) If $n = 100$, how many ways are there to choose exactly 40 students out of n that go for a trip to Riga?

There are three different buses (red, yellow and blue), each bus fits 40 passengers.

- (b) If $n = 30$, how many ways are there to arrange the students into the buses (it is allowed that some buses are empty)?
- (c) Now, $n = 100$, and it is known that some 40 students will be seated in the red bus, 32 students will be seated in the yellow bus, and the rest will be seated in the blue bus. How many ways to distribute the students between the buses are there?

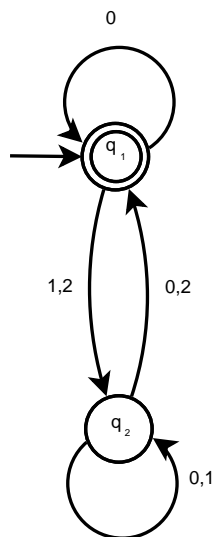
Five students from the group were assigned to carry 20 identical guidebooks.

- (d) How many ways to distribute the guidebooks between these five students are there, if each of the students carries at least one guidebook? (These five students are already chosen.)
- (e) In a group of $n = 100$ students, it is known that during the holiday season:
- 50 students went to Tallinn;
 - 45 students went to Riga;
 - 33 students went to Vilnius;
 - 11 students went to Tallinn and to Riga;
 - 9 students went to Tallinn and to Vilnius;
 - 3 students went to Tallinn, to Riga and to Vilnius;
 - 2 students did not go to either of the three capitals.

How many students went to Riga and Vilnius during the holiday season?

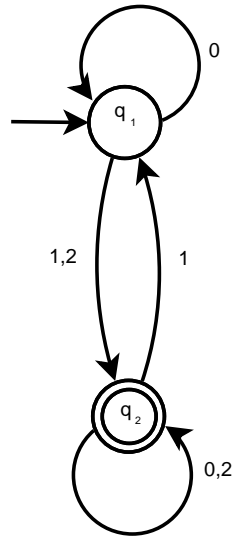
Question 2 (22 points).

Convert the following nondeterministic finite automaton into an equivalent deterministic automaton. Show all the steps in the conversion process.



Question 3 (22 points).

By using the method, which was shown in the class, construct a regular expression for the language \mathcal{L} defined by the following deterministic finite automaton:



Show all the steps in the algorithm.

Question 4 (26 points).

Let $\Sigma = \{0, 1\}$ be an alphabet.

- (a) Is the following language regular? Justify your answer.

$$\mathcal{L}_1 = \left\{ www \mid w \in \Sigma^* \text{ is a string of length less or equal to } 17 \right\}.$$

- (b) Prove that the following language is not regular:

$$\mathcal{L}_2 = \left\{ www \mid w \in \Sigma^* \right\}.$$

