

Midterm examApril 15th, 2019

Student name: _____**Student ID:** _____

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 110 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable). Show all important steps in your solution.
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (35 points).

In the parliament of A Far Far Land there are 101 elected representatives.

- (a) In how many ways can the parliament form a committee of five members out of its 101 representatives?
- (b) In how many ways can the parliament form three committees, each committee has five members, each member can serve on any number of committees?
- (c) In how many ways can the parliament form three committees, the committees consist of 5, 10 and 21 members, each member can serve only on at most one committee?
- (d) To celebrate a national holiday, 12 identical chocolate bars are given to five members of the committee, such that each member receives at least one chocolate bar. In how many ways can this be done?
- (e) The parliament forms three committees: the committee for science, the committee for culture and the committee for sports. It is known that:
 - the committee for science has 45 members;
 - the committee for culture has 32 members;
 - the committee for sports has 29 members;
 - 7 members participate in the committees both for science and for culture;
 - 8 members participate in the committees both for culture and for sports;
 - 3 members participate in all three committees;
 - 16 members of the parliament do not participate in any of the three committees.

How many members of the parliament participate in both the committee for science and the committee for sports?

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Question 2 (20 points).

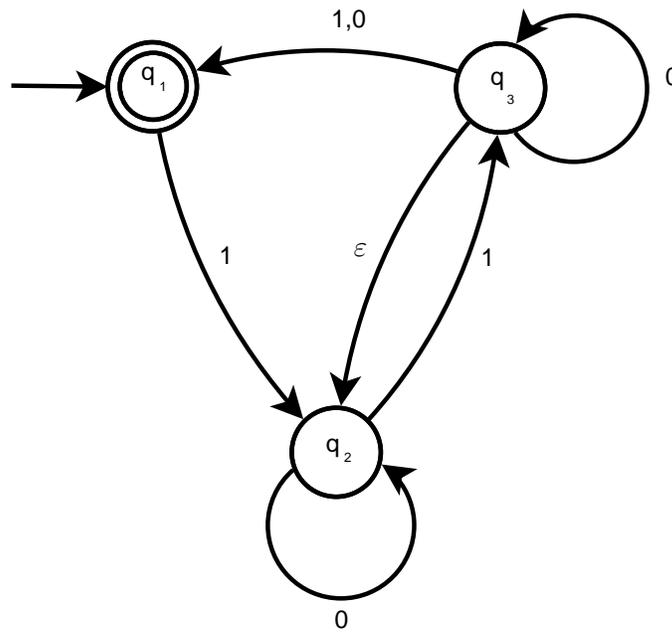
Construct a finite automaton (either deterministic or non-deterministic) that recognizes the language described by the following regular expression: $\varepsilon \cup (101)^+$. Give a justification why your automaton is correct.

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Question 3 (20 points).

Convert the following nondeterministic finite automaton into an equivalent deterministic automaton. Show all the steps in the conversion process.



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Question 4 (35 points).

Let $\Sigma = \{0, 1\}$ be an alphabet. For a string $w \in \Sigma^*$ define N_{01} , N_{10} and N_{11} to be the number of times the pair of symbols “01”, “10” and “11”, respectively, appears in w .

Example: Take $w = 010101110$. Then, $N_{01} = 3$, $N_{10} = 3$ and $N_{11} = 2$.

(a) Is the following language regular? Justify your answer.

$$\mathcal{L}_1 = \left\{ w \mid N_{01} = N_{10} \right\}.$$

(b) Is the following language regular? Justify your answer.

$$\mathcal{L}_2 = \left\{ w \mid N_{01} = N_{11} \right\}.$$

