MTAT.05.125: Introduction to Theoretical Computer Science

University of Tartu

## Midterm exam

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Student name: \_\_\_\_\_

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- 1. This exam contains 10 pages. Check that no pages are missing.
- 2. It is possible to collect up to 110 points. Try to collect as many points as possible.
- 3. Justify and prove all your answers (where applicable).
- 4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
- 5. Any printed and written material is allowed in the class. No electronic devices are allowed.
- 6. Exam duration is 1 hour 40 minutes.
- 7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (35 points).

A group of 10 children came to a shop that sells balloons. The balloons come in four colors: yellow, red, green and blue. Assume that there are infinitely many balloons of each color.

- (a) Each child selects one balloon. In how many ways the children can choose their balloons?
- (b) Each child selects three balloons. In how many ways the children can choose their balloons?
- (c) Each child selects at least one balloon in such a way that all his/her balloons are of different colors. In how many ways the children can choose their balloons?
- (d) Assume now that the shop has only 3 yellow, 2 red, 1 green and 4 blue balloons (10 balloons in total). Each child gets one balloon. How many possibilities are there?
- (e) Assume now that the shop has 18 identical yellow balloons, which are distributed between the 10 children. In how many ways this can be done?
- (f) Similar to (e), but in addition it is known that no child gets more than 4 balloons. In how many ways this can be done?

Question 2 (20 points).

Convert the following nondeterministic finite automaton into an equivalent deterministic automaton. Show all the steps in the conversion process.



Question 3 (20 points).

Construct a regular expression for the language  ${\cal L}$  defined by the following deterministic finite automaton:



Show all the steps in the algorithm.

Question 4 (35 points).

Let  $\Sigma = \{0, 1, 2\}$  be an alphabet. Consider strings  $w \in \Sigma^*$ .

(a) Is the following language regular? Justify your answer.

 $\mathcal{L}_1 = \left\{ \begin{array}{c} w \end{array} \middle| \ \text{the sum of digits within any substring of } w \text{ of length 3 is 3} \end{array} \right\}.$ For example,  $01201 \in \mathcal{L}_1$ , but  $01112 \notin \mathcal{L}_1$ .

(b) Let  $n\geq 3$  be an integer. Is the following language regular? Justify your answer.

 $\mathcal{L}_2 = \left\{ w \mid \text{ the sum of digits within any substring of } w \text{ of length } n \text{ is at least } n \right\}.$ 

(c) Prove that the following language is not regular.

 $\mathcal{L}_3 = \left\{ \begin{array}{c} w \end{array} \middle| \text{ the sum of the first } n \text{ digits in } w \text{ is larger or equal} \\ \text{ to the sum of the last } n \text{ digits (for all } n \ge 1) \right\}.$ 

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