

Midterm exam

Instructors: Vitaly Skachek, Yauhen Yakimenka, Reimo Palm

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Student name: _____

Student ID: _____

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

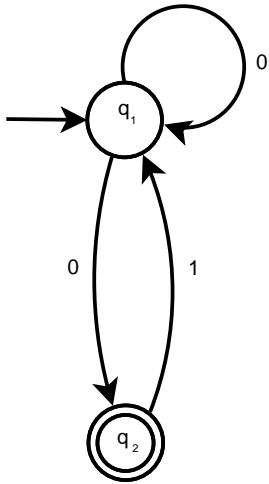
Question 1 (48 points).

In the movie festival there are 7 different movies, each movie is screened on a different day.

- (a) Robert is a big fan of cinema, so he would like to see all the movies. However, since he is a poor student, his budget allows him to buy only 5 tickets (and this is with the student discount!). Help Robert to count how many ways are there to choose 5 movies out of 7.
- (b) Martin wants to choose an arbitrary number of movies to see (i.e., Martin can choose to see no movie, or to see any single movie, or any two movies, or any three movies, . . . , or all seven movies). How many choices are there?
- (c) A group of 14 students wants to see the movies. Each student chooses any number of movies. The choices of students are independent of each other. How many choices are there?
- (d) A group of 14 students wants to see the movies, and each movie is seen by exactly two students. How many choices are there?
- (e) A group of 14 students wants to see the movies. Each student chooses 3 movies. Additionally, it is known that each movie is seen by at least one student. How many choices are there?

Question 2 (24 points).

Convert the following nondeterministic finite automaton into equivalent deterministic automaton. Show all the steps in the conversion process.



Question 3 (24 points).

Construct a deterministic finite automaton $\mathcal{M} = (Q, \Sigma, \delta, q_0, F)$, which recognizes the language defined by the regular expression: $(010)^*$. Specify all the ingredients Q, Σ, δ, q_0 and F of \mathcal{M} .

Question 4 (24 points).

Prove that the following language is not regular:

$$\mathcal{L} = \{ 0^m 1^n \mid m \leq 2n + 5, m, n \in \mathbb{N} \} .$$

