MTAT.05.125: Introduction to Theoretical Computer Science **University of Tartu**

Midterm exam

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Student name: _____

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1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 110 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (35 points).

A group of 10 children came to a shop that sells balloons. The balloons come in four colors: yellow, red, green and blue. Assume that there are infinitely many balloons of each color.

(a) Each child selects one balloon. In how many ways the children can choose their balloons?

(b) Each child selects three balloons. In how many ways the children can choose their balloons?

(c) Each child selects at least one balloon in such a way that all his/her balloons are of different colors. In how many ways the children can choose their balloons?

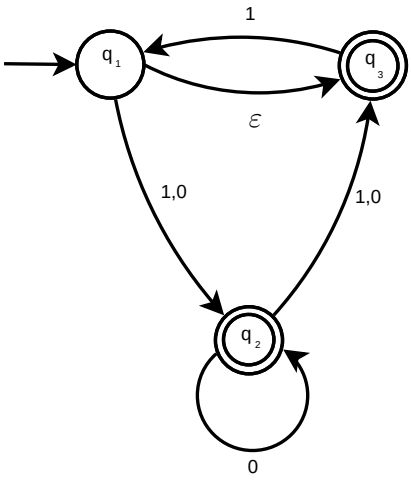
(d) Assume now that the shop has only 3 yellow, 2 red, 1 green and 4 blue balloons (10 balloons in total). Each child gets one balloon. How many possibilities are there?

(e) Assume now that the shop has 18 identical yellow balloons, which are distributed between the 10 children. In how many ways this can be done?

(f) Similar to (e), but in addition it is known that no child gets more than 4 balloons. In how many ways this can be done?

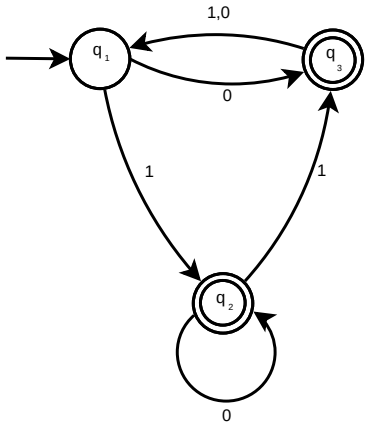
Question 2 (20 points).

Convert the following nondeterministic finite automaton into an equivalent deterministic automaton. Show all the steps in the conversion process.



Question 3 (20 points).

Construct a regular expression for the language $\mathcal{L}$ defined by the following deterministic finite automaton:



Show all the steps in the algorithm.

Question 4 (35 points).

Let $\Sigma = \{0, 1, 2\}$ be an alphabet. Consider strings $w \in \Sigma^*$.

(a) Is the following language regular? Justify your answer.

$$\mathcal{L}_1 = \left\{ w \;\middle|\; \text{the sum of digits within any substring of } w \text{ of length 3 is 3} \right\}.$$

For example, $01201 \in \mathcal{L}_1$, but $01112 \notin \mathcal{L}_1$.

(b) Let $n \geq 3$ be an integer. Is the following language regular? Justify your answer.

$$\mathcal{L}_2 = \left\{ w \;\middle|\; \text{the sum of digits within any substring of } w \text{ of length } n \text{ is at least } n \right\}.$$

(c) Prove that the following language is not regular.

$$\mathcal{L}_3 = \left\{ w \;\middle|\; \text{the sum of the first } n \text{ digits in } w \text{ is larger or equal to the sum of the last } n \text{ digits (for all } n \geq 1) \right\}.$$