

Final exam

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Student name: _____

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1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 110 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 2 hours.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (30 points).

Define the language

$$\mathcal{L}_{\text{diff}} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } \mathcal{L}(A) \setminus \mathcal{L}(B) = \{0^m 1^n \mid m, n \geq 0\} \} .$$

Show that $\mathcal{L}_{\text{diff}}$ is a decidable language.

Hint: you can use the fact that $\mathcal{L}_{\text{DFA-EQ}}$ is a decidable language (shown in the class), where

$$\mathcal{L}_{\text{DFA-EQ}} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } \mathcal{L}(A) = \mathcal{L}(B) \} .$$

Question 2 (30 points).

Define the language

$$\mathcal{L}_\infty = \{\langle \mathcal{M} \rangle \mid \mathcal{M} \text{ is a Turing machine and } \mathcal{L}(\mathcal{M}) \text{ has infinite cardinality}\} .$$

Show that \mathcal{L}_∞ is an undecidable language.

Hint: for example, you can use a reduction from the language \mathcal{L}_{TM} ,

$$\mathcal{L}_{\text{TM}} = \{\langle \mathcal{M}, w \rangle \mid \mathcal{M} \text{ is a Turing machine and } \mathcal{M} \text{ accepts the input string } w\} .$$

Question 3 (20 points).

Define a language SAT-1000-LITERALS:

$$\text{SAT-1000-LITERALS} = \left\{ \langle \phi \rangle \mid \begin{array}{l} \phi \text{ is a satisfiable CNF-formula} \\ \text{with at most 1000 different literals} \end{array} \right\}.$$

Is SAT-1000-LITERALS $\in \mathcal{P}$? Justify your answer.

Question 4 (30 points).

Definition: an r -coloring of vertices of an undirected finite graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a function $c : \mathcal{V} \rightarrow \{1, 2, \dots, r\}$ (that assigns “colors” $1, 2, \dots, r$ to the vertices in \mathcal{V}) such that for any two adjacent vertices $u, v \in \mathcal{V}$ it holds $c(u) \neq c(v)$.

Define a language r -COLORING-WITH-PARAMETERS:

$$r\text{-COLORING-WITH-PARAMETERS} = \left\{ \langle \mathcal{G}, r, n_1, n_2, \dots, n_r \rangle \mid \mathcal{G} \text{ is an undirected graph} \right. \\ \left. \text{that has an } r\text{-coloring with } n_i \text{ vertices colored in color } i \text{ for all } i = 1, 2, \dots, r \right\}.$$

In this question, you will show that r -COLORING-WITH-PARAMETERS is \mathcal{NP} -complete.

- (a) Prove that r -COLORING-WITH-PARAMETERS $\in \mathcal{NP}$.
- (b) Prove that r -COLORING-WITH-PARAMETERS is \mathcal{NP} -hard.

Hint: you can use a polynomial-time reduction from VERTEX-COVER to r -COLORING-WITH-PARAMETERS. Do not forget to show that the reduction is correct and polynomial-time.

