Homework assignment 6

Due date: December 4th, 2017

It is possible to collect up to 110 points in this homework.

1. Define the language

   \[ L_7 = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ does not contain any strings of length } 7n \text{ for } n \geq 0 \} . \]

   Show that \( L_7 \) is a decidable language.

2. Define the language

   \[ L_{1^*} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ contains infinitely many strings } w \text{ in } 1^* \} . \]

   Show that \( L_{1^*} \) is a decidable language.

3. Show that the language \( L \) is Turing-recognizable if and only if \( L \leq_M \text{HALT} \).

   Reminder:

   \[ \text{HALT} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine and } M \text{ halts on the input string } w \} . \]

4. Define the language

   \[ L_{17} = \{ \langle M \rangle \mid M \text{ is a Turing machine and } |L(M)| = 17 \} . \]

   Prove that \( L_{17} \) is an undecidable language.

   **Hint:** for example, you can use reduction from the language \( L_{\text{TM}} \). Assume that there exists a Turing machine that decides \( L_{17} \). Show how to construct a Turing machine that decides \( L_{\text{TM}} \).