Homework Assignment 5

Due date: November 20th, 2017

It is possible to collect up to 110 points in this homework.

1. Let $w$ be a string over the alphabet $\Sigma = \{0, 1\}$. Denote by $\#_0(w)$ and $\#_1(w)$ the number of zeros and of ones in $w$, respectively. Give an implementation-level description of the Turing machine that decides the following language over $\Sigma$:

$$L = \{ w \mid \#_0(w) = \#_1(w) + 2 \}.$$

2. Prove that any regular language $L$ is Turing-decidable.

3. Definition: let $L$ be a language over an alphabet $\Sigma$. Define a complementary language $\overline{L}$ as

$$\overline{L} = \{ w \mid w \in \Sigma^* \text{ and } w \notin L \}.$$

Definition: let $L_1$ and $L_2$ be two languages over an alphabet $\Sigma$. Then,

$$L_1 \oplus L_2 = \{ w \mid w \in \Sigma^* \text{ and } w \text{ is in exactly one of the languages } L_1, L_2 \}.$$

Prove the following statements.

(a) If $L$ is a Turing-decidable language then $\overline{L}$ is also Turing-decidable;

(b) If $L_1$ and $L_2$ are two Turing-decidable languages then $L_1 \oplus L_2$ is also Turing-decidable.

Hint: assume that there exist Turing machines $M_1$ and $M_2$ that decide $L_1$ and $L_2$, respectively. Use them to construct Turing machine that decides the language $L_1 \oplus L_2$.

4. In this question, we define a Turing machine with stay in place head operation instead of left. It is defined similarly to an ordinary Turing machine, but the transition function is

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, S\}.$$

At each moment of the computation, the machine can move its head to the right (denoted by $R$) or to let it stay in the same position (denoted by $S$). Show that this variant of Turing machine is not equivalent to the usual version. Prove that the class of languages recognized by these machines is a class of regular languages.

Hint: assume that $M$ is a Turing machine with stay in place head operation instead of left. Observe that on the given input the head of $M$ never moves backwards. Construct a deterministic finite automaton that recognizes the same language as $M$. 