It is possible to collect up to 110 points in this homework.

1. For each of the following deterministic finite automata $M = (Q, \Sigma, \delta, q_0, F)$ describe all the ingredients $Q, \Sigma, \delta, q_0$ and $F$. What language is recognized by each $M$?

   (a) 
   
   (b)

2. For two languages $L_1$ and $L_2$, define the difference $L_1 \setminus L_2$ as follows:

   $$L_1 \setminus L_2 = \{ w \mid w \in L_1 \text{ and } w \notin L_2 \} .$$

   Show that if $L_1$ and $L_2$ are two regular languages over the alphabet $\Sigma$, then $L_1 \setminus L_2$ is regular over the same alphabet.

   Example: if $L_1 = \{001, 110, 11\}$ and $L_2 = \{110, 00\}$, then $L_1 \setminus L_2 = \{001, 11\}$.

3. Let $\Sigma = \{0, 1, 2\}$. For a given string denote by $n_1$ and $n_2$ the number of symbols ‘1’ and ‘2’ in it, respectively.

   (a) Construct a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, which recognizes the language of all strings with $n_1 + 2 \cdot n_2$ divisible by 5.

   (b) Construct a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, which accepts the language of all strings, where each substring of length four contains symbol ‘2’.

   In both parts describe all the ingredients $Q, \delta, q_0$ and $F$. 
4. (a) Describe all the ingredients $Q, \Sigma, \delta, q_0$ and $F$ of the following nondeterministic finite automaton $\mathcal{N} = (Q, \Sigma, \delta, q_0, F)$. What language does it recognize?

(b) Convert $\mathcal{N}$ into an equivalent deterministic automaton.