

Lahendused

October 27, 2021

1.

$$\begin{aligned}P &= P(J = 1, M = 1, A = 1, B = 0, E = 0) \\ &= P(J = 1|A = 1)P(M = 1|A = 1)P(A = 1|B = 0, E = 0)P(B = 0)P(E = 0) \\ &= 0.9 \cdot 0.7 \cdot 0.001 \cdot 0.999 \cdot 0.998 \\ &= 6.28 \times 10^{-4}\end{aligned}$$

2. Küsitud: $P = P(B = 1|A = 1, E = 0, M = 1, J = 1)$. Seda saab esitada, vastavalt tingliku tõenäosuse definitsioonile kui

$$P(B = 1|A = 1, E = 0, M = 1, J = 1) = \frac{P(A = 1, E = 0, M = 1, B = 1, J = 1)}{P(A = 1, E = 0, M = 1, J = 1)}.$$

Siin

$$P(A = 1, E = 0, M = 1, J = 1) = \sum_{b \in \{0,1\}} P(A = 1, E = 0, M = 1, J = 1, B = b).$$

Samas, võrgu kujust saame, et

$$\begin{aligned}P(A = a, B = b, E = e, M = m, J = j) \\ = P(J = j|A = a)P(M = m|A = a)P(A = a|B = b, E = e)P(B = b)P(E = e)\end{aligned}$$

st

$$\begin{aligned}P(A = 1, E = 0, M = 1, B = 1, J = 1) \\ = P(J = 1|A = 1)P(M = 1|A = 1)P(A = 1|B = 1, E = 0)P(B = 1)P(E = 0)\end{aligned}$$

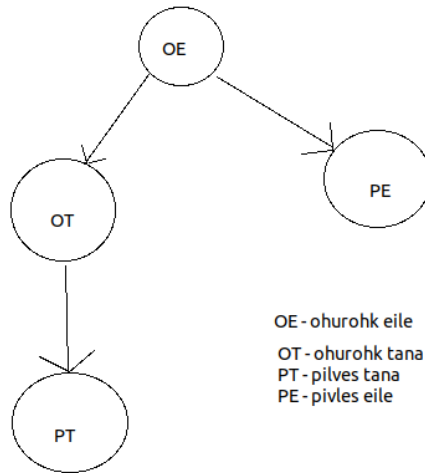
ja

$$\begin{aligned}P(A = 1, E = 0, M = 1, J = 1) \\ = P(J = 1|A = 1)P(M = 1|A = 1)P(E = 0)[P(A = 1|B = 1, E = 0)P(B = 1) \\ + P(A = 1|B = 0, E = 0)P(B = 0)].\end{aligned}$$

Seega kokku võttes taanduvad sulgude ees olevad (B -d mittesisaldavad) liikmed ära ja

$$\begin{aligned}P(B = 1|A = 1, E = 0, M = 1, J = 1) \\ = \frac{P(A = 1|B = 1, E = 0)P(B = 1)}{P(A = 1|B = 1, E = 0)P(B = 1) + P(A = 1|B = 0, E = 0)P(B = 0)} \\ = \frac{0.94 \cdot 0.001}{0.94 \cdot 0.001 + 0.001 \cdot 0.999} \approx 0.48.\end{aligned}$$

3. Pildi peal on selle ja järgmise ülesande Bayesi võrk.



4. Küsitud on $P(OT = madal|PE = 1, PT = 1)$. Selle jaoks saab samamoodi toimida nagu teises ülesandes. Edaspidi tähistame m - madal, k - kõrge.

$$P(OT = m|PE = 1, PT = 1) = \frac{P(OT = m, PE = 1, PT = 1)}{P(PE = 1, PT = 1)}.$$

Analoogselt saab summeerides üle mittesisalduvate muutujate leida need ühistõenäosused. Saame

$$\begin{aligned}
 &P(OT = madal, PE = 1, PT = 1) \\
 &= \sum_{oe \in \{m, k\}} P(OT = m, PE = 1, PT = 1, OE = oe) \\
 &= \sum_{oe \in \{m, k\}} P(PT = 1|OT = m)P(OT = m|OE = oe)P(PE = 1|OE = oe)P(OE = oe) \\
 &= P(PT = 1|OT = m) \sum_{oe \in \{m, k\}} P(OT = m|OE = oe)P(PE = 1|OE = oe)P(OE = oe) \\
 &= 0.6 \cdot (0.3 \cdot 0.6 \cdot 0.4 + 0.2 \cdot 0.4 \cdot 0.6) \\
 &= 0.12
 \end{aligned}$$

ning

$$\begin{aligned}
 P(PE = 1, PT = 1) &= \sum_{oe \in \{m, k\}} \sum_{ot \in \{m, k\}} P(OT = ot, PE = 1, PT = 1, OE = oe) \\
 &= \sum_{oe \in \{m, k\}} \sum_{ot \in \{m, k\}} P(PT = 1|OT = ot)P(OT = ot|OE = oe)P(PE = 1|OE = oe)P(OE = oe) \\
 &= \sum_{ot \in \{m, k\}} P(PT = 1|OT = ot) \sum_{oe \in \{m, k\}} P(OT = ot|OE = oe)P(PE = 1|OE = oe)P(OE = oe) \\
 &= 0.6 \cdot (0.3 \cdot 0.6 \cdot 0.4 + 0.2 \cdot 0.4 \cdot 0.6) + 0.4 \cdot (0.7 \cdot 0.6 \cdot 0.4 + 0.8 \cdot 0.4 \cdot 0.6) \\
 &= 0.3144.
 \end{aligned}$$

Seega küsitud tõenäosus on

$$P(OT = m|PE = 1, PT = 1) = \frac{0.12}{0.3144} = 0.3817.$$