Text Algorithms (4AP)
Lecture 2: Exact pattern matching

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Exact pattern matching

- $S = s_1 s_2 \ldots s_n$ (text) $|S| = n$ (length)
- $P = p_1 p_2 .. p_m$ (pattern) $|P| = m$
- Σ - alphabet $|\Sigma| = c$
- Does S contain P?
  - Does S = S' P S" fo some strings S' ja S"?
  - Usually m << n and n can be (very) large

Find occurrences in text

Algorithms

- Brute force
- Knuth-Morris-Pratt
- Karp-Rabin
- Shift-OR, Shift-AND
- Boyer-Moore
- Factor searches

Animations

- http://www-igm.univ-mlv.fr/~lecroq/string/

- **EXACT STRING MATCHING ALGORITHMS Animation in Java**
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Brute Force

Identify the first mismatch!

Question:

- Problems of this method? ☹
- Ideas to improve the search? ☺

Brute force

Algorithm Naive

Input: Text S[1..n] and
pattern P[1..m]

Output: All positions i, where P occurs in S

```
for( i=1 ; i <= n-m+1 ; i++ )
   for ( j=1 ; j <= m ; j++ )
      if( S[i+j-1] != P[j] ) break ;
if ( j > m ) print i ;
```

```
attempt 1:
gcatcgcagagagtatacagtacg
GCAg....

attempt 2:
gcatcgcagagagtatacagtacg
 g.......

attempt 3:
gcatcgcagagagtatacagtacg
  g.......

attempt 4:
gcatcgcagagagtatacagtacg
   g.......

attempt 5:
gcatcgcagagagtatacagtacg
    g.......

attempt 6:
gcatcgcagagagtatacagtacg
     GCAGAGAG

attempt 7:
gcatcGCAGAGAGtatacagtacg
      g.......
```

NaiveSearch

```
1  function NaiveSearch(string s[1..n], string sub[1..m])
2  for i from 1 to n-m+1
3      for j from 1 to m
4          if s[i+j-1] ≠ sub[j]
5              jump to next iteration of outer loop
6      return i
7  return not found
```

C code

```
int bf_2( char* pat, char* text , int n ) /* n = textlen */
{
   int m, i, j ;
   int count = 0 ;
   m = strlen(pat);

   for ( i=0 ; i + m <= n ; i++) {

         for( j=0; j < m && pat[j] == text[i+j] ; j++) ;

         if( j == m )
                  count++ ;
   }

   return(count);
}
```

C code

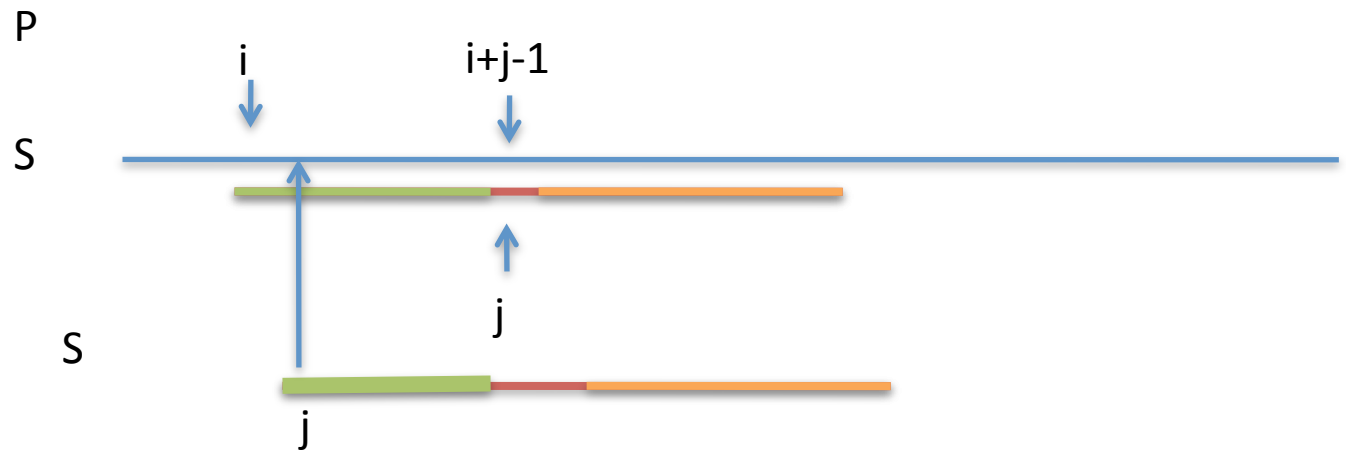
```
int bf_1( char* pat, char* text )
{
        int m ;
        int count = 0 ;
        char *tp;

        m = strlen(pat);
        tp=text ;

        for( ; *tp ; tp++ ) {
                if( strncmp( pat, tp, m ) == 0 ) {
                        count++ ;
               }
        }

        return( count );
}
```

Main problem of Naive



- For the next possible location of P, check again the same positions of S

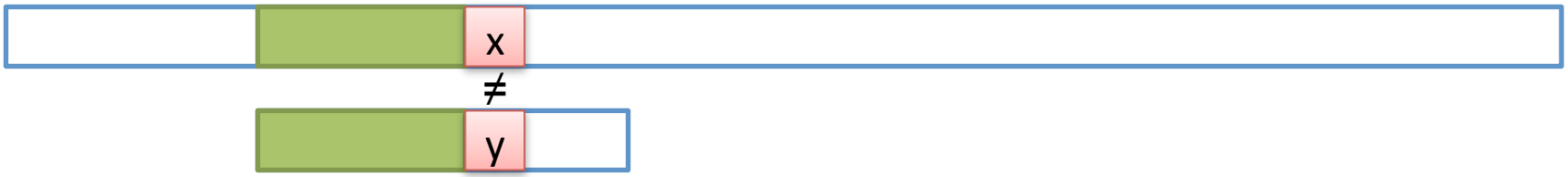
Goals

- Make sure only a constant nr of comparisons/ operations is made for each position in S
  - Move only from left to right in S
  - How?
  - After a test of S[i] <> P[j] what do we now ?

Knuth-Morris-Pratt

D. Knuth, J. Morris, V. Pratt:
Fast Pattern Matching in strings.
SIAM Journal on Computing 6:323-350, 1977.

- Make sure that no comparisons “wasted”

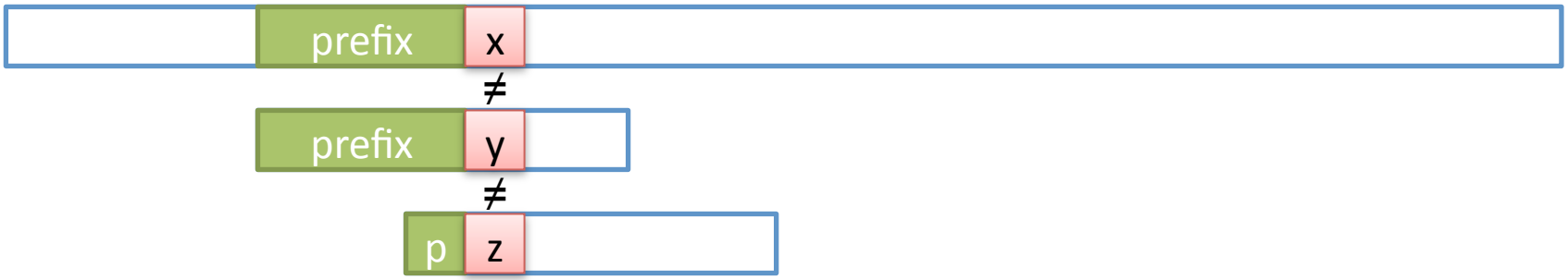


- After such a mismatch we already know exactly the values of green area in S !

Knuth-Morris-Pratt

D. Knuth, J. Morris, V. Pratt:
Fast Pattern Matching in strings.
SIAM Journal on Computing 6:323-350, 1977.

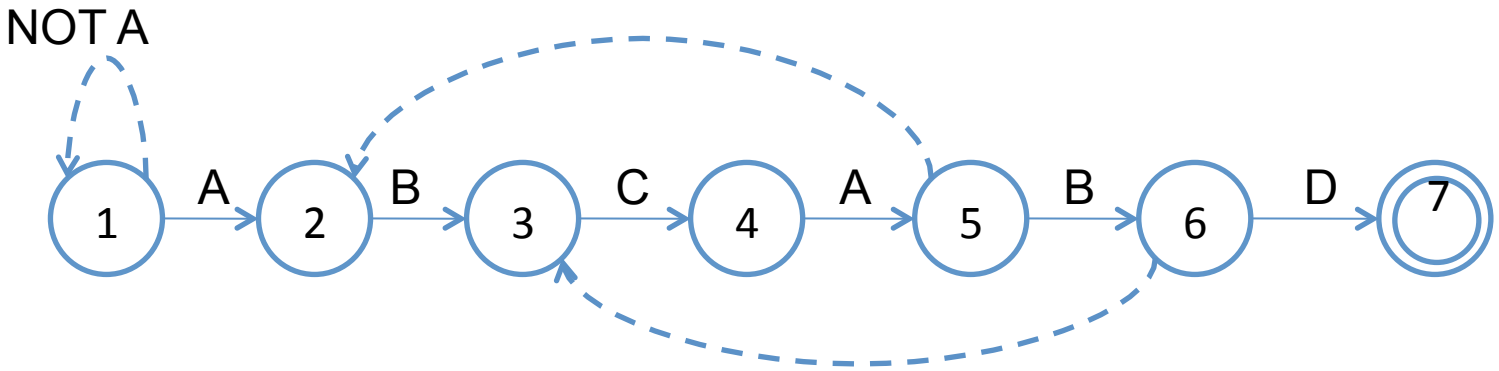
- Make sure that no comparisons "wasted"



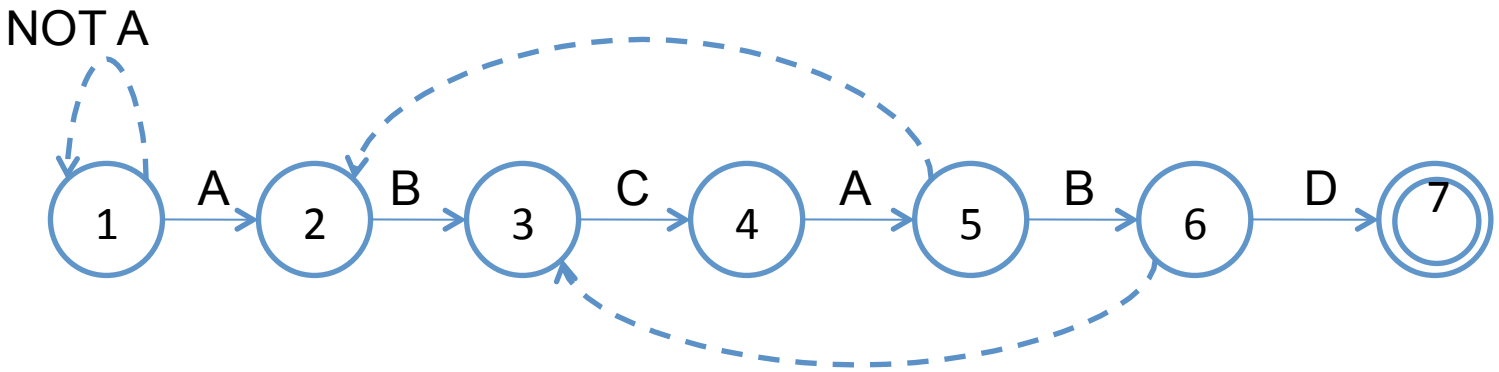
- P – longest suffix of any prefix that is also a prefix of a pattern
- Example: ABCABD

ABCABD

Automaton for ABCABD



Automaton for ABCABD



Fail:	0	1	1	1	2	3	1

Pattern:	A	B	C	A	B	D
	1	2	3	4	5	6

KMP matching

Input: Text S[1..n] and pattern P[1..m]

Output: First occurrence of P in S (if exists)

```
i=1; j=1;
initfail(P) // Prepare fail links
repeat
   if j==0 or S[i] == P[j]
   then i++ , j++ // advance in text and in pattern
   else j = fail[j] // use fail link
until j>m or i>n
if j>m then report match at i-m
```

Initialization of fail links

Algorithm: KMP_Initfail

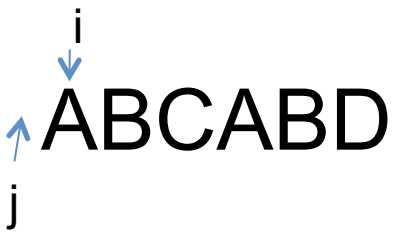
Input: Pattern P[1..m]

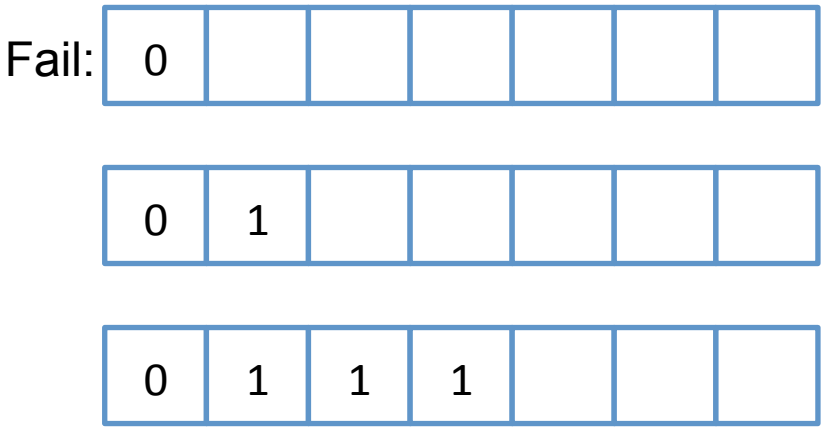
Output: fail[] for pattern P

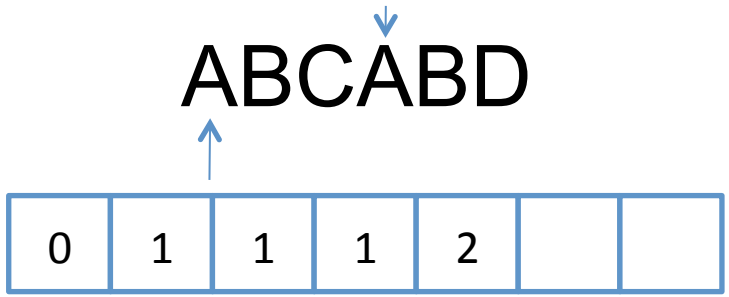
```
i=1, j=0 , fail[1]= 0
repeat
  if j==0 or P[i] == P[j]
  then i++ , j++ , fail[i] = j
  else j = fail[j]
until i>=m
```

Initialization of fail links

```
i=1, j=0 , fail[1]= 0
repeat
   if j==0 or P[i] == P[j]
   then i++ , j++ , fail[i] = j
   else j = fail[j]
until i>=m
```







Analysis of time complexity

- At every cycle either i and j increase by 1
- Or j decreases (j=fail[j])

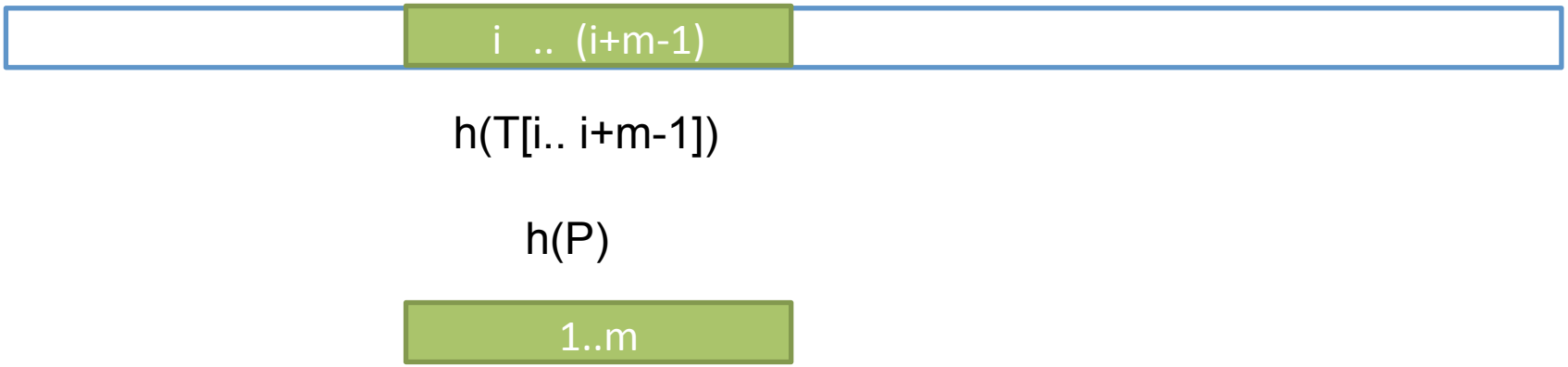
- i can increase n (or m) times
- Q: How often can j decrease?
  - A: not more than nr of increases of i

- **Amortised analysis**: O(n) or O(m)

Karp-Rabin

R.Karp and M. Rabin: Efficient randomized pattern-matching algorithms.
IBM Journal of Research and Development 31 (1987), 249-260.

- Compare in O(1) a hash of P and S[i..i+m-1]



- Goal: O(n).
- f(h(T[i.. i+m-1]) -> h(T[i+1..i+m])) = O(1)

Karp-Rabin

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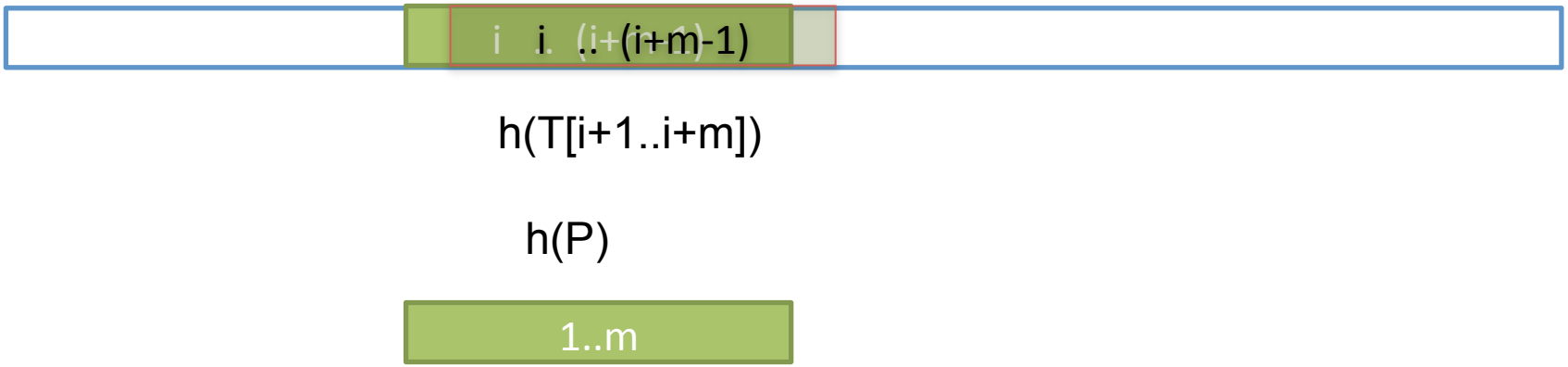
- Compare in O(1) a hash of P and S[i..i+m-1]



- Goal: O(n).
- f(h(T[i.. i+m-1]) -> h(T[i+1..i+m])) = O(1)

Hash

- “Remove” the effect of T[i] and “Introduce” the effect of T[i+m] – in O(1)
- Use base |Σ| arithmetics and treat charcters as numbers
- In case of hash match – check all m positions
- Hash collisions => Worst case O(nm)

Let’s use numbers

- T= 57125677
- P= 125 (and for simplicity, h=125)

- H(T[1])=571
- H(T[2])= (571-5*100)*10 + 2 = 712

- H(T[3]) = (H(T[2]) – ord(T[1])*10m)*10 + T[3+m-1]

hash

- c – size of alphabet
- HSi = H(S[i..i+m-1])
- $H(S[i+1 .. i+m]) = (HSi – ord(S[i])* c^{m-1}) * c + ord(S[i+m])$
- Modulo arithmetic – to fit value in a word!

- $hash(w[0 .. m\text{-}1]) = (w[0]*2^{m-1} + w[1]*2^{m-2} + \cdots + w[m\text{-}1]*2^{0}) \bmod q$

Karp-Rabin

```
Input: Text S[1..n] and pattern P[1..m]
Output: Occurrences of P in S
1. c=20; /* Size of the alphabet, say nr. of aminoacids */
2. q = 33554393 /* q is a prime */
3. cm = c^(m-1) mod q
4. hp = 0 ; hs = 0
5. for i = 1 .. m do hp = ( hp*c + n(p[i]) ) mod q // H(P)
6. for i = 1 .. m do hs = ( hp*c + n(s[i]) ) mod q // H(S[1..m])
7. if hp == hs and P == S[1..m] report match at position

8. for i=2 .. n-m+1
9.    hs = ( (hs - n(s[i-1])*cm) * c + n(s[i+m-1]) mod q
10.  if hp == hs and P == S[i..i+m-1]
11.       report match at position i
```

More ways to ensure O(O(1) * n) ?

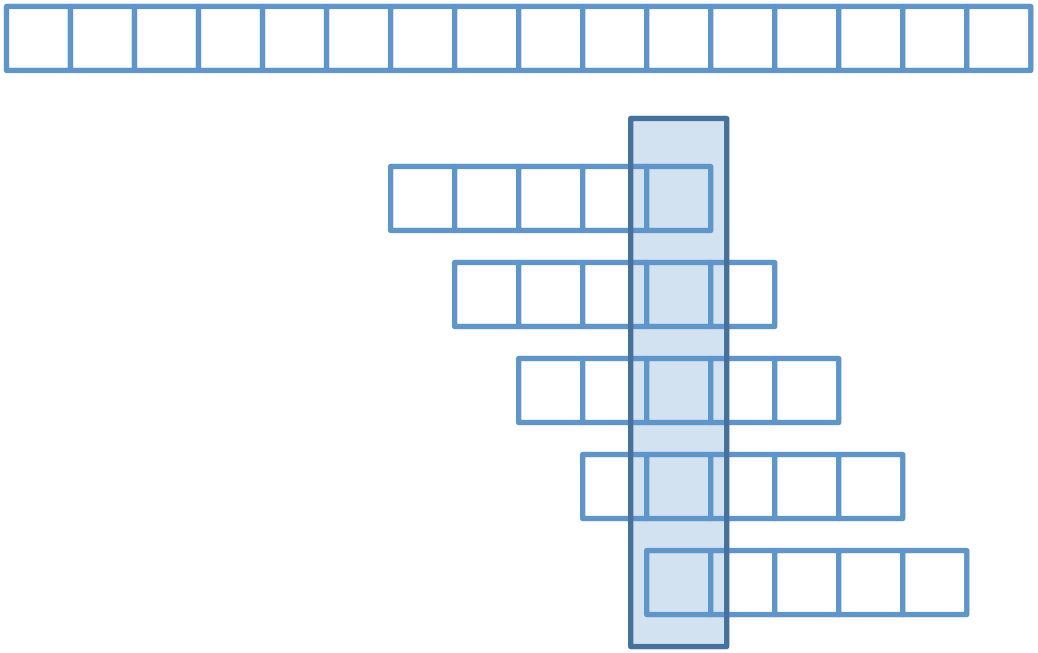
Shift-AND / Shift-OR

- **Ricardo Baeza-Yates , Gaston H. Gonnet**
  A new approach to text searching
  Communications of the ACM October 1992, Volume 35 Issue 10
  [ACM Digital Library:http://doi.acm.org/10.1145/135239.135243] [DOI]
- PDF

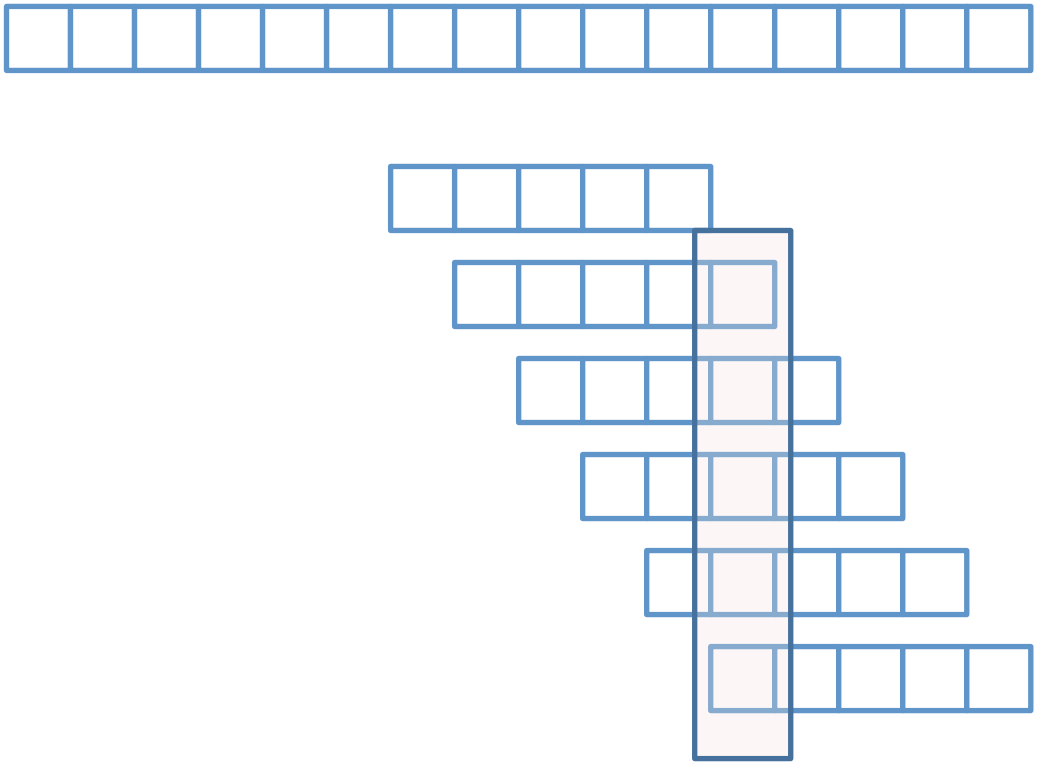
Bit-operations

- Maintain a set of all prefixes that have so far had a perfect match
- On the next character in text update all previous pointers to a new set
- Bit vector: for every possible character

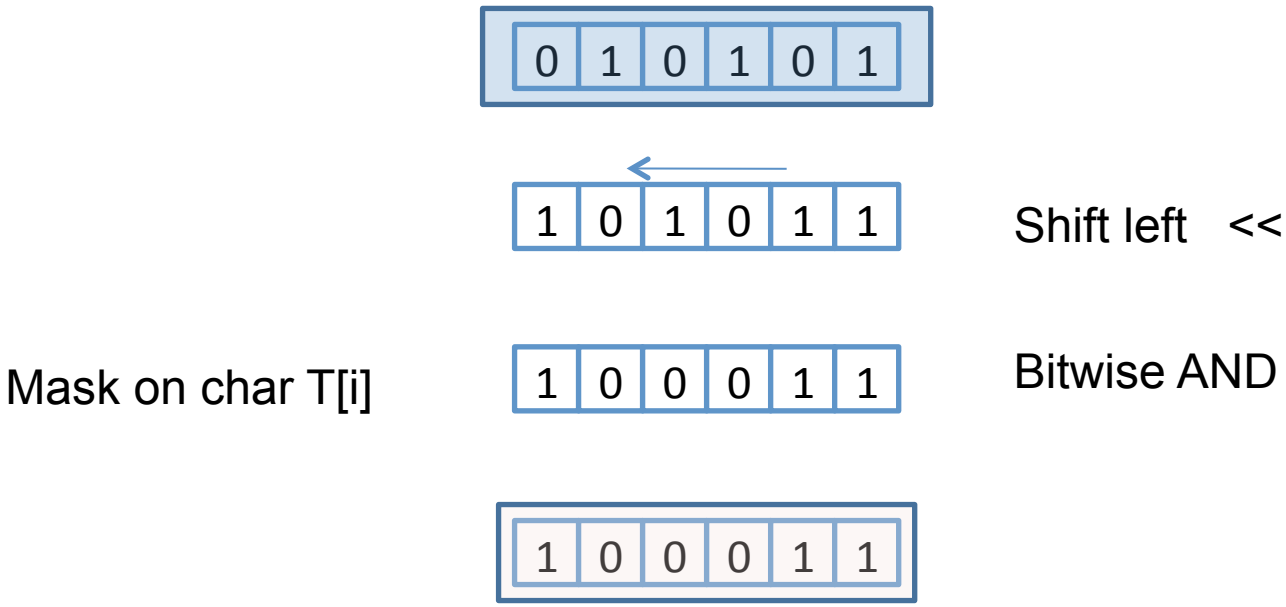
State: which prefixes match?



Move to next:



Track positions of prefix matches



Vectors for every char in Σ

- P=aste

```
a  s  t  e  b  c  d  ..  z
1  0  0  0  0  ...
0  1  0  0  0  ...
0  0  1  0  0  ...
0  0  0  1  0  ...
```

- T= lasteaed

```
l a s t e a e d
0 1
0 0
0 0
0 0
```

- T= lasteaed

```
l a s t e a e d
0 1 0
0 0 1
0 0 0
0 0 0
```

- T= lasteaed

l	a	s	t	e	a	e	d
0	1	0	0	0	1		
0	0	1	0	0	0		
0	0	0	1	0	0		
0	0	0	0	**1**	0		

- T= lasteaed

	l	a	s	t	e	a	e	d
	0	1	0	0	0	1		
	0	0	1	0	0	0		
	0	0	0	1	0	0		
	0	0	0	0	**1**	0		

Summary

Algorithm	Worst case	Ave. Case	Preprocess
Brute force	O(mn)	O(n * (1+1/|Σ|+..)	
Knuth-Morris-Pratt	O(n)	O(n)	O(m)
Rabin-Karp	O(mn)	O(n)	O(m)
Boyer-Moore		O(n/m) ?	
BM Horspool			
Factor search			
Shift-OR	O(n)	O(n)	O(m|Σ|)

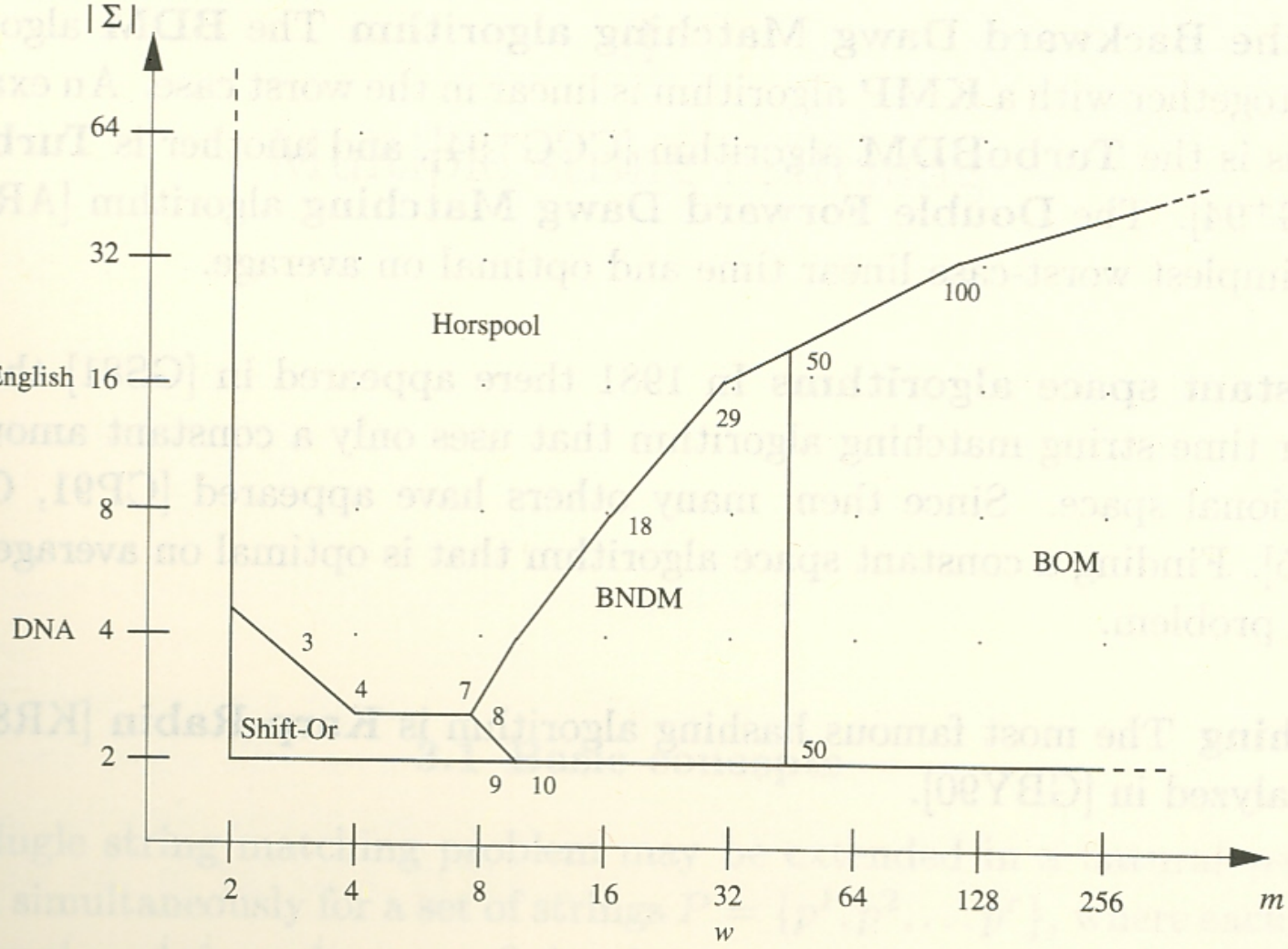


Fig. 2.22. Map of experimental efficiency for different string matching algorithms.